Learning with the Speed of Mumbai and the Tradition of Kota

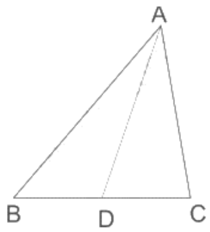
Max Marks: 40 | **Date: 00. 00. 2022**

Time: 2 HRS | **Seat No. ________**

S.S.C GEOMETRY: SAMPLE PAPER -1 ANSWER

Q1. A. Choose the correct alternative **(4)**

(i) In Fig BD = 8, BC = 12, B – D – C then $\frac{A(\Delta ABC)}{A(\Delta ABD)}$ = ?



(a) 2 : 3 (b) 3 : 2
(c) 5 : 3 (d) 3 : 4

(ii) Two Circles having diameter 8 cm and 6cm touch each other internally, the distance between their centres is _______ cm

(a) 2 (b) 14 (c) 7 (d) 1

(iii) If Point P is midpoint of segment joining point A (-4,2) and point B (6,2) then co-ordinates of pare ____

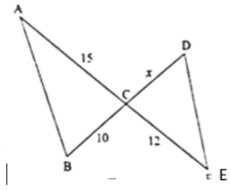
(a) (-1, 2) (b) (1,2) (c) (1, -2) (d) (-1, -2)

(iv) If ∠ A = 30^0 then tan 2A = ?

(a) 1 (b) 0 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

Q1 B. Solve the following sub question: **(4)**

(i) In the figure, Δ ABC ~ Δ EDC
AC = 15, BC = 10, CE = 12 find x



Sol. Δ ABC ~ Δ EDC

$\therefore \frac{AC}{EC} = \frac{BC}{CD}$ .... (c.s.s.t)

$\therefore \frac{15}{12} = \frac{10}{x}$

$\therefore x = \frac{10 \times 12}{15}$

$\therefore x = 8$

(ii) Observe the triplet (4,5,8). State whether it is a Pythagorean triplet or out.

Sol. The given triplet is not a Pythagorean triplet

(iii) The Volume of a cube is 1000 cm^3. Find the side of the cube.

Sol. Let the side of the cube be l cm.

The volume of a cube = l^3

$\therefore 1000 = l^3$

$(10)^3 = (l)^3$

$\therefore l = 10$ cm

(iv) If $\cos\theta = \frac{\sqrt{3}}{2}$, then find the value of acute angle θ

Sol $\cos\theta = \frac{\sqrt{3}}{2}$ ……………….. (1)

$\cos 30^0 = \frac{\sqrt{3}}{2}$ ………… (2)

From 1 & 2, the value of acute angle θ is 30^0

Q2A Complete any two of three activities. (6)

(i) Height of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of areas of these triangles

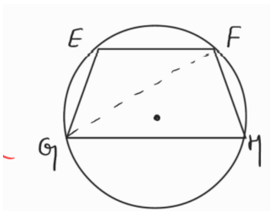
Sol Let the area of smaller triangle be A, and the area of greater triangle be A_2.
Let their heights be h_1 and h_2 respectively.

$\therefore \frac{A_1}{A_2} = \frac{h_1^2}{h_2^2}$ ……..By the theories of areas of similar triangle

$= \frac{6^2}{9^2}$

$= \frac{4}{9}$

(ii) In figure, chord EFl l Chord GH. Prove that chord EG ≅ Chord FH. Fill in the blanks and write the proof.



Sol. Proof : Draw Seg GF

∠EFG ……………… Alternate angle theorem ……… (1)

∠EFG = ½ m (arc E.G.) (Inscribed angle theorem) (2)

∠FGH = ½ m (arc F.H) (Inscribed angle theorem) (3)

∴ m (arc EG) = m (arc FH) (By 1,2, and 3)

Chord EG ≅ Chord FH …… (Corresponding Chords of congruent arcs).

(iii) Complete the following Activity to draw a tangent at a point on the circle.

Draw a circle of radius 3.5 c with O as centre

↓

Take any point P on the Circle

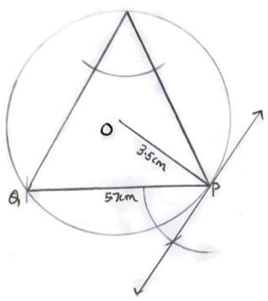
↓

Draw Chord PQ and an Inscribed ∠PRQ.

↓

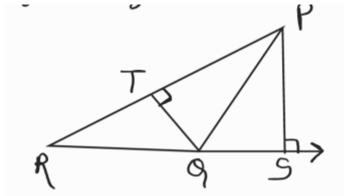
Construct a Congruent angle to ∠PRQ at point P to draw tangent at point P.

Sol.



Q2.B By Solve any Four of the following Sub questions. **(10)**

(i) In the adjoining figure,
Seg PS ⊥ Seg RQ,
Seg QT ⊥ Seg PR.
If RQ = 6, PS = 6 and PR = 12, Then find QT



Sol. Given : Seg PS ⊥ seg RQ, Seg QT ⊥ Seg PR

RQ = 6, PS = 6, PR =12

To find : QT

A (Δ PQR) = $\frac{1}{2}$ x RQ x PS ……. as RQ ⊥ PS

= $\frac{1}{2}$ x 6 x 6

= 18 unit^2 ………..(1)

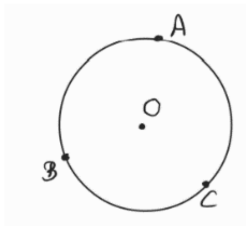
Also, A (Δ PQR) = $\frac{1}{2}$ PR x QT …… as PR ⊥ QT

18 = $\frac{1}{2}$ x 12 x QT From 1

$\frac{18 \times 2}{12}$ = QT

3 = QT

(ii) A,B, C are any Points on the circle with centre O.
Write the names of all arcs formed due to these point.
If m (arc BC) = 110^0 and m (arc AB) = 125^0, find m (arc ABC)



Sol. Names of acrs.

arc AB , arc BC, arc AC, arc ABC, arc ACB, arc BAC.

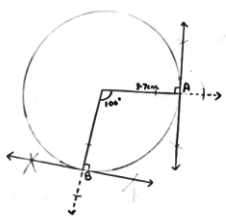
Also m (arc AB) + m (arc BC) = m (arc ABC)

125^0 + 110^0 = m (arc ABC)

235^0 = m (arc ABC)

(iii) Draw a Circle with Centre P. Draw an arc A of 100^0 measure. Draw tangents to the Circle at point A and Point B

Sol.



(iv) Measure of an arc of a circle is 80^0 and its radius is 18 cm. Find the length of the arc . (π = 3.14)

Sol. Radius (r) = 18 cm.

Central angle (θ) = 80^0.

Length of the arc (l) = $\frac{\theta}{360}$ x 2π r

= $\frac{80^0}{360^0}$x 2 x 3 .14 x 18

$= 25.12$ cm

$\therefore$ Length of the arc is 25.12 cm.

(v) If $\tan\theta = \frac{21}{20}$, find $\sec\theta$

Sol. Given that $\tan\theta = \frac{21}{20}$

Using trigonometric Identify

$1 + \tan^2\theta = \sec^2\theta$

$\therefore \quad 1 + \left(\frac{21}{20}\right)^2 = \sec^2\theta$

$\therefore \quad 1 + \frac{441}{400} = \sec^2\theta$

$\therefore \quad \frac{400 + 441}{400} = \sec^2\theta$

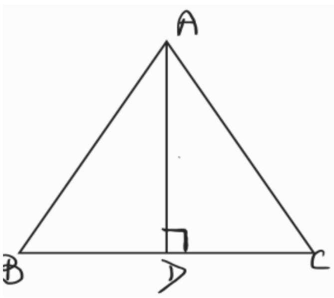
$\therefore \quad \frac{841}{400} = \sec^2\theta$

$\therefore \quad \frac{\sqrt{841}}{\sqrt{400}} = \sec\theta$

$\frac{29}{20} = \sec\theta$

Q3. (A) Complete any one out of two activities: (6)

(i) Δ ABC is an Equilateral triangle. Seg AD $\perp$ Seg BC such that B-D-C. Prove $AD^2 = 3BD^2$ by completing the following activity.



In Δ ABD ?

$\angle ADB = \boxed{90^0}$ .... (Given)

$\angle B = \boxed{60^0}$ ..... Angle of an Equilateral triangle

$\therefore \angle BAD = 30^0$ ..... Remaining angle of Δ ABD

$\therefore \Delta ABD$ is a $\boxed{30^0 - 60^0 - 90^0}$ Triangle

$AD = \boxed{\frac{\sqrt{3}}{2}} AB$ .... (side opposite to 60^0) ....(1)

and $BD = \boxed{\frac{1}{2}} AB$ .... (side opposite to 30^0)

$AB = 2BD$ ..... (2)

Substituting Value of AB from (2) in (1)

$AD = \frac{\sqrt{3}}{2} \times 2BD$

$\therefore AD = \boxed{\sqrt{3}} BD$

$\therefore AD^2 = 3\,BD^2$ ..,, (Squaring both the sides)

(ii) If the points P (-4, -2) , Q (-3, -7), R (3,-2) and 5 (2,3) are joined serially. Find the type of quadrilateral PQRS by completing the following activity.

Sol

Slope of line PQ = $\frac{-7-\boxed{-2}}{-3-\boxed{-4}} = \boxed{-5}$

Slope of line QR = $\frac{\boxed{-2}-(-7)}{\boxed{3}-(-3)} = \boxed{5/6}$

Slope of line QR = $\frac{3-(-2)}{2-3} = \boxed{-5}$

Slope of line PS = $\frac{3-(-2)}{2-(-4)} = \boxed{5/6}$

In □ PQRS, slopes of opposite sides are $\boxed{\text{equal}}$

∴ □ PQRs, is a $\boxed{\text{parallelogram}}$

Q3.B Solve any two of the following sub questions (12)

(i) The dimensions of cuboid are 44 cm, 21 cm, 12 cm. It is melted and a cone of height 24 cm is made. Find the radius of its base.

Sol Here, l = 44 cm, b = 21 cm, h = 12 cm

Volume of the Cuboid = l x b x h

= (44 x 21 x 2) cm^3

For the cone, h = 24 cm

Volume of the Cone = $\frac{1}{3}\pi r^2 h$

$= \left(\frac{1}{3} \times \frac{22}{7} \times r2 \times 24\right) cm^3$

Now, Volume of the Cone = Volume of the Cuboid

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 44 \times 21 \times 12$

∴ $r^2 = \frac{44 \times 21 \times 12 \times 7 \times 3}{22 \times 24}$

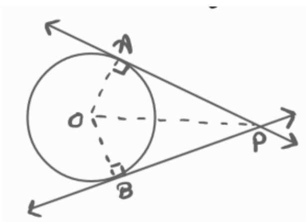
∴ r^2 = 441

∴ r = 21 cm

∴ Radius of the base of the cone is 21 cm

(ii) Prove the following statement, "Tangent Segment drawn from an external point to a circle are congruent."

Sol



Given : O is the centre of the Circle and P is point in the exterior of the Circle.

A and B are the points of contact of the two tangents from P to the circle

To Prove : PA = PB

Construction : Draw seg OA, Seg OB and Seg OP.

Proo : In Δ AOP and Δ BOP

Seg OA ≅ Seg QB Radii of the same circle.

Seg OD ≅ Seg OD Common side

m ∠OAP ≅ m ∠OBP = 90^0….. Tangent theorem

∴ Δ AOP ≅ Δ BOP …. Hypotenuse side test of congruency

∴ Seg AP ≅ Seg BP ….. (c.s.c.t)

(iii) Verify that points P (-2,2), Q (2,2) and R (2,7) are vertices of a right angled triangle.

Let P ≡ (x_1, y_1) ≡ (-2,2)

Q ≡ (x_2, y_2) ≡ (2,2)

R ≡ (x_3, y_3) ≡ (2,7)

D (P,Q) $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[2-(-2)]^2 + (2-2)^2}$

$= \sqrt{(4)^2 + (0)^2}$

$= \sqrt{16 + 0}$

∴ d (P,Q) = PQ = $\sqrt{16}$ ⇒ ∴ $PQ^2 = 16$ …….(i)

D (Q,R) $= \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$= \sqrt{(2-2)^2 + (7-2)^2}$

$= \sqrt{(0)^2 + (5)^2}$

$= \sqrt{0 + 25}$

∴ d (Q, R) = QR = $\sqrt{25}$ ⇒ ∴ $QR^2 = 25$ ………. (ii)

d (P,R) $= \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$= \sqrt{[2-(-2)]^2 + (7-2)^2}$

$= \sqrt{(4)^2 + (5)^2}$

$= \sqrt{16 + 25}$

∴ d(P,R) = PR = $\sqrt{41}$ ⇒ ∴ $PR^2 = 41$ ……… (iii)

From 1,2 and 3

16 + 25 = 41

∴ $PQ^2 + QR^2 = PR^2$

∴ By Converse of Pythagoras theorem, Δ PQR is a right angled triangle

∴ P (-2,2), Q (2,2) and R (2,7) are the vertices of a right angled triangle.

(iv) Δ RST ~ Δ XYZ. In Δ RST, RS = 4.5 cm ∠RST = 40^0, ST = 5.7 cm. Construct Δ XYZ Such that $\frac{RS}{XY} = \frac{3}{5}$

Sol. Analysis

Δ RST ~ Δ XYZ .... (Given)

$\frac{RS}{XY} = \frac{ST}{YZ} = \frac{3}{5}$ ..... Corresponding sides of Similar Triangle.

$\therefore \frac{RS}{XY} = \frac{3}{5}$, $\frac{ST}{YZ} = \frac{3}{5}$

$\therefore \frac{4.5}{XY} = \frac{3}{5}$, $\frac{5.7}{YZ} = \frac{3}{5}$

$\therefore$ XY = $\frac{4.5 \times 5}{3}$, YZ = $\frac{5.7 \times 5}{3}$

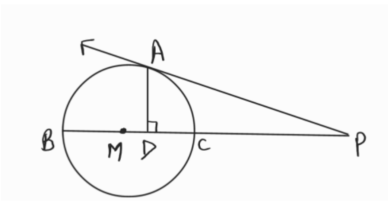
$\therefore$ XY = 7.5 CM , YZ = 9.5 CM

$\angle$ RST = $\angle$ XYZ = 40^0 Corresponding angles of Similar Triangles.

Thus In Δ XYZ, XY = 7.5 cm
YZ = 9.5 cm
And $\angle$ XYZ = 40^0

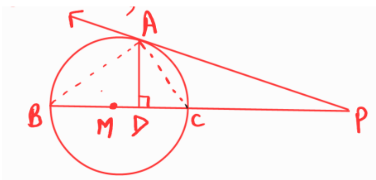
Q4. Solve any two of the following subquestion **(12)**

(i) In the figure, BC is diameter of the Circle with Centre M. PA is tangent at A from P which is a point on line BC. AD $\perp$ BC. Prove that DP^2 = BP X CP – BD x CD.



Sol. BC is a diameter of the Circle, M is the Centre of the Circle. PA is tangent at A from P which is a point on line BC, AD $\perp$ BC

Construction Join AC and AB



Proof:

Line PA is tangent and line PB is a secant.

$\therefore$ PA^2 = PC x PB ....(i) [By Tangent secant Property]

In Δ PAD , $\angle$ADB = 90^0 .... Given

BY Pythagoras theorem

$PA^2 = AD^2 + DP^2$ ....(ii)

Substituting (ii) in (i) we get,

$DP^2 + AD^2$ = BP x CP

DP^2 = BP x CP - AD^2 ....(iii)

Now In Δ BAC, BC is the diameter.

$\therefore$ $\angle$BAC = 90^0 $\therefore$ Angle Inscribed In a Semicircle is a right angle.

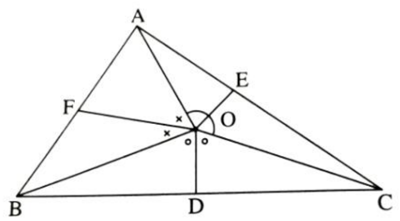
Seg AD $\perp$ BC $\therefore$ Given

AD^2 + BD x DC .... (iv) By property of Geometric Mean

Substituting (iv) in (iii), We get

DP^2 = BP x CP – BD x CD

(ii) Q is any point in the Interior of ΔABC. Bisectors of ∠AOB, ∠BOC and ∠AOC intersect side AB, Side BC, Side AC in F, D and E respectively.
Prove:
BF x AE x CD = AF x CE x BD



Sol. Proof:
In Δ AOB
ray OF bisect ∠AOB …………(Given)
$\frac{AO}{OB} = \frac{AF}{FB}$ (1) ……….. Property of an angle bisector of a triangle

In Δ BOC
Ray OD bisects ∠BOC …. (given)
$\frac{OB}{OC} = \frac{BD}{DC}$ … (2) Property of an angle bisector of a triangle

In Δ AOC,
Ray OE bisects ∠AOC ……. (given)
$\frac{OC}{OA} = \frac{CE}{AE}$ … (3) ….. Property of an angle Bisector of a triangle

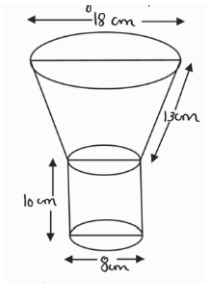
Multiply (1), (2), (3), we get

$\frac{AO}{OB} \times \frac{OB}{OC} \times \frac{OC}{OA} = \frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{AE}$

$\therefore 1 = \frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{AE}$

∴ AF X BD X CE = BF X AE X CD

(iii) The funnel of tine sheet Consist of a cylindrical portion 10 cm long attached to a frustrum of a cone, If diameter of the top and bottom of the frustrum is 18 cm and 8 cm respectively and the slant height of the frustrum of cone is 13 cm. Find the surface area of the tin required to make the funnel.



Sol. Diameter of a Cylindrical Portion = 8cm
Its radius (r_1) = 4cm
Height of a Cylindrical portion (h) = 10 cm

Diameter of the top of the frustrum = 18 cm
∴ Radius of the top of frustrum (r_2) = 9 cm
Slant height (l) of frustrum = 13 cm

Also radius of the bottom of frustrum = radius of Cylindrical portion

Surface area of tin required = Curved Surface are of Cylindrical portion + Curved Surface area of Frustrum portion .

$= \quad 2\pi r_1 h + \pi (r_1 + r_2)\, l$

$= \quad 2\pi \times 4 \times 10 + \pi (4 + 9) \times 13$

$= \quad 80\pi + 169\pi$

$= \quad 249\pi \text{ cm}^2$

Q5. **Solve any One of the following subquestion.** **(6)**

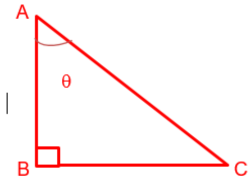
(i) Draw a right angled Δ ABC such that $\angle$ ABC = 90^0. Consider $\angle$BAC = θ then,

(a) Write $\sin\theta$ and hence $\sin^2\theta$

(b) Write $\cos\theta$ and hence $\cos^2\theta$

(c) Simplify $\sin^2\theta + \cos^2\theta$ and write your answer

Sol.



In Δ ABC $\angle$BAC = θ, AC is Hypotenuse. By Pythagoras Theorem

$\therefore AC^2 = AB^2 + BC^2$ ....(1)

(a) $\text{Sin}\,\theta = \dfrac{\text{Opposite Side of } \theta}{\text{Hypotenuse}}$

$= \dfrac{BC}{AC}$

On squaring both the sides we get,

$\therefore \text{Sin}^2\theta = \dfrac{BC^2}{AC^2}$

(b) $\text{Cos}\,\theta = \dfrac{\text{Adjacent Side of } \theta}{\text{Hypotenuse}}$

$= \dfrac{AB}{AC}$

On squaring both the Sides we get,

$\therefore \text{Cos}^2\theta = \dfrac{AB^2}{AC^2}$

(c) Consider,

$\text{Sin}^2\theta + \text{Cos}^2\theta = \dfrac{BC^2}{AC^2} + \dfrac{AB^2}{AC^2}$

$= \dfrac{BC^2 + AB^2}{AC^2}$

$= \dfrac{AC^2}{AC^2}$ ……. From 1

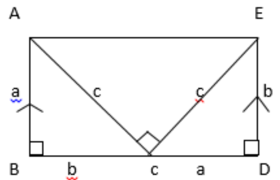
$\therefore \text{Sin}^2\theta + \text{Cos}^2\theta = 1$

(ii) Two Congruent right angled triangles (ΔABC and ΔCDE) and another isosceles right angled triangle (ΔACE) whose congruent sides are equal to the hypotenuse of the two congruent right angled triangle are taken. After joining these triangles, the figure so formed is a trapezium. (□ ABDE)

(a) Find the area of trapezium.

(b) Find the sum of area of the three right angled triangles.

(c) Equate area of trapezium with the sum of areas the three right angled triangles and hence Prove $C^2 = a^2 + b^2$.



Ans. (a) Area of Trapezium = $\frac{1}{2}$ (sum of length of parallel x height side)

$= \frac{1}{2}$ (a + b) (a + b)

$= \frac{(a+b)^2}{2}$

$= \left(\frac{a^2+2ab+b^2}{2}\right)$ unit ^{2}

(b) Area of Right angled triangle = $\frac{1}{2}$ x Product of Perpendicular sides

Sum of areas of 3 triangles = $\frac{1}{2}$ x a x b + $\frac{1}{2}$ x c x c + $\frac{1}{2}$ x a x b

$= \frac{1}{2} ab + \frac{1}{2} c^2 + \frac{1}{2} ab$

$= ab + \frac{1}{2} c^2$

$= \left(\frac{2ab+c^2}{2}\right)$ unit ^{2}

(c) Area of Trapezium = Sum of areas of 3 triangles

$$\frac{a^2+2ab+b^2}{2} = \frac{2ab+c^2}{2}$$

$\therefore a^2 + b^2 = c^2$